Dependencies between Analyses and Transformations in the Middle-End of a Compiler

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Analyze to compile? Compile to analyze?

Whom is this talk for?

- Compiler designers?
- Analyzer developers?
- Who is here today anyway?

What is abstract interpretation?

Winter 81-82, Metz, ante Feautrier ☺:

- A model for the environment (possibly)

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- A model for the commands (possibly)
- Automatic and correct derivation of the model(s) from the program
- Automatic solving of the model equations

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- A model for the values and the state: *The abstract domain*
- A model for the commands (possibly)
- Automatic and correct derivation of the model(s) from the program
- Automatic solving of the model equations
- Decidability: over-approximations

What's missing from a compiler viewpoint?

- Changing code after each transformation pass

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- Multiple analyses

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- Changing code after each transformation pass
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- Dependant analyses
- Under-approximations or exactness for non-monotonous equations
- ...

Running example

```
int main()
{
   float a[100][100];
   int n = read_input(100, a);
   int p = init_parameter(n, a);
   compute(p, n, a);
   write_output(n, a);
}
```

```
void compute(int p, int n, float a[n][n])
{
   float t[n];
   int k = n;
   int i, j;
   for(i = 0; i <= n-1; i += 1) {
      k--;
      t[k] = a[i][i];
      for(j = 0; j <= n-1; j += 1)
         if (p>0)
            a[i][j] /= t[k];
   }
}
```

Running example: optimized and parallelized

```
void compute(int p, int n, float a[n][n])
{
   float t[n];
   int k = n;
   int i, j;
   for(i = 0; i <= n-1; i += 1) {
      k--;
      t[k] = a[i][i];
      for(j = 0; j <= n-1; j += 1)
         if (p>0)
            a[i][j] /= t[k];
   }
}
```

```
void compute(int p, int n, float a[n][n])
{
   int i, j;
   //PIPS generated variable
   float __scalar__0;
#pragma omp parallel for private(j,__scalar__0)
   for(i = 0; i <= 99; i += 1) {
      __scalar__0 = a[i][i];
#pragma omp parallel for
      for(j = 0; j <= 99; j += 1)
         a[i][j] /= __scalar__0;
   }
}
```

Control simplification

```
void compute(int p, int n, float a[n][n])
{
   float t[n];
   int k = n;
   int i, j;
   for(i = 0; i <= n-1; i += 1) {
      k--;
      t[k] = a[i][i];
      for(j = 0; j <= n-1; j += 1)
         if (p>0)
            a[i][j] /= t[k];
   }
}
```

```
void compute(int p, int n, float a[n][n])
{
   float t[n];
   int k = n;
   int i, j;
   for(i = 0; i <= n-1; i += 1) {
      k--;
      t[k] = a[i][i];
      for(j = 0; j <= n-1; j += 1)
         a[i][j] /= t[k];
   }
}
```

- Condition c, `p>0`, is always true under precondition P

$$\{\sigma | P(\sigma) \wedge \mathcal{E}(\bar{c}, \sigma)\} = \emptyset$$

- Similar use of P for zero and one-trip loops, for infinite loops

Preconditions: where do they come from?

```
//  P() {n==100, 1<=p}
void compute(int p, int n, float a[n][n])
{
   float t[n];
   int k = n;
   int i, j;

   for(i = 0; i <= n-1; i += 1) {
      k--;
      t[k] = a[i][i];
      for(j = 0; j <= n-1; j += 1)
//  P(i,j,k) {i+k==99, n==100, 0<=i, i<=99, 0<=j, j<=99, 1<=p}
         if (p>0)
            a[i][j] /= t[k];
   }
}
```

```
int main()
{
   float a[100][100];
   int n = read_input(100, a);
//  P(n) {n==100}
   int p = init_parameter(n, a);
//  P(n,p) {n==100, 1<=p}
   compute(p, n, a);
   write_output(n, a);
}
```

```
//  T(init_parameter) {1<=init_parameter}
int init_parameter(int n, float a[n][n])
{
   int p;
//  T(p) {p<=0, p#init<=p, p#init<=0}
   while (p<=0)
//  T(p) {p==p#init+1}
      p++;
//  T(init_parameter) {init_parameter==p}
   return p;
}
```

Induction variable substitution

```
void compute(int p, int n, float a[n][n])
{
   float t[n];
   int k = n;
   int i, j;
   for(i = 0; i <= n-1; i += 1) {
      k--;
      t[k] = a[i][i];
      for(j = 0; j <= n-1; j += 1)
         a[i][j] /= t[k];
   }
}
```

```
void compute(int p, int n, float a[n][n])
{
   float t[n];
   int k = n;
   int i, j;
   for(i = 0; i <= n-1; i += 1) {
      k = -(i+-100)-1;
      t[-(-99+i)] = a[i][i];
      for(j = 0; j <= n-1; j += 1)
         a[i][j] /= t[-(-99+i)];
   }
}
```

Variable k can be susbtituted in statement S with precondition P within a loop of index i if P defines a mapping from $\sigma(i)$ to $\sigma(k)$:

$$v \longrightarrow \{v' | \exists \sigma \in P \ \sigma(i) = v \wedge \sigma(k) = v'\}$$

Constant Propagation

```
void compute(int p, int n, float a[n][n])
{
   float t[n];
   int k = n;
   int i, j;
   for(i = 0; i <= n-1; i += 1) {
      k = -(i+-100)-1;
      t[-(-99+i)] = a[i][i];
      for(j = 0; j <= n-1; j += 1)
         a[i][j] /= t[-(-99+i)];
   }
}
```

```
void compute(int p, int n, float a[n][n])
{
   float t[100];
   int k = 100;
   int i, j;
   for(i = 0; i <= 99; i += 1) {
      k = -i+99;
      t[-i+99] = a[i][i];
      for(j = 0; j <= 99; j += 1)
         a[i][j] /= t[-i+99];
   }
}
```

An expression e can be substituted under precondition P if:

$$|\{v \in Val | \exists \sigma \in P \;\; v = \mathcal{E}(e, \sigma)\}| = 1$$

Iterative transformer and precondition equations

- For a sequence of statements S_i at step n:

$$\begin{aligned} T^n_{S_i} &= \mathcal{T}(S_i, P^{n-1}_{S_i}) \wedge P^{n-1}_{S_i} \\ P^n_{S_i} &= T^n_{S_{i-1}} \circ P^n_{S_{i-1}} \\ P^0_{S_i} &= Id \end{aligned}$$

- Useful for some (rare) nested loops...
- No convergence guarantee
- Not used for running example ☺

After Allen&Kennedy Parallelization

```
void compute(int p, int n, float a[n][n])
{
   float t[100];
   int k = 100;
   int i, j;
   for(i = 0; i <= 99; i += 1) {
      k = -i+99;
      t[-i+99] = a[i][i];
      for(j = 0; j <= 99; j += 1)
         a[i][j] /= t[-i+99];
   }
}
```

```
void compute(int p, int n, float a[n][n])
{
   float t[100];
   int k = 100;
   int i, j;
   for(i = 0; i <= 99; i += 1)
      k = -i+99;
#pragma omp parallel for
   for(i = 0; i <= 99; i += 1)
      t[-i+99] = a[i][i];
#pragma omp parallel for
   for(i = 0; i <= 99; i += 1)
#pragma omp parallel for
      for(j = 0; j <= 99; j += 1)
         a[i][j] /= t[-i+99];
}
```

Dependence system for two references in statements S_1 and S_2:

$$\sigma_1(i) \prec \sigma_2(i) \wedge T_{S_1,S_2}(\sigma_1,\sigma_2) \wedge P_{S_1}(\sigma_1) \wedge P_{S_2}(\sigma_2) \wedge \ldots$$

Coarse Grain Parallelization

- Assumes convex array regions R and W:

  $R, W \in Id \times \Sigma \rightarrow \mathcal{P}(\Phi)$
- Direct parallelization of a loop using convex array regions and Bernstein's condition on body B iterations:

  $\forall v \in Id$

  $$\{\phi | \forall \sigma, \sigma' \in P_B \phi \in R_{B,v}(\sigma) \wedge \phi \in W_{B,v}(\sigma') \wedge \sigma(i) < \sigma'(i)\} = \emptyset$$
- Beyond Bernstein's conditions using IN and OUT array regions:

  $\forall v \in Id$

  $\{\phi | \forall \sigma, \sigma' \in P_B \ \phi \in OUT_{B,v}(\sigma) \wedge \phi \in IN_{B,v}(\sigma') \wedge \sigma(i) < \sigma'(i)\} = \emptyset$

Array privatization (different example)

```
void compute(int p, int n, float a[n][n])
{
   int t[n];
   int k = n, i, s;
   for(i = 0; i <= n-1; i += 1) {
      int j;
      for(j = 0; j <= n-1; j += 1)
         t[j] = a[i][j]+j;

      for(j = 0; j <= n-1; j += 1)
         if (p>0)
            a[i][j] = t[j];
   }
}
```

```
void compute(int p, int n, float a[n][n])
{
   int t[100];
   int k = 100, i, s;
#pragma omp parallel for private(t[100])
   for(i = 0; i <= 99; i += 1) {
      int j;
#pragma omp parallel for
      for(j = 0; j <= 99; j += 1)
         t[j] = a[i][j]+j;

#pragma omp parallel for
      for(j = 0; j <= 99; j += 1)
         a[i][j] = t[j];
   }
}
```

- An array k is privatizable in a loop l with body B if $IN_{B,k} = OUT_{B,k} = \emptyset$
- $IN_{B,k}$ is the set of elements of k whose input values are used in B
  $$IN(S_1; S_2) = IN(S_1) \cup IN(S_2) \circ T(S_1) - W(S_1)$$
- $OUT_{B,k}$ is the set of elements of k whose output values are used by the continuation of B.

After scalarization (back to the running example)

```
void compute(int p, int n, float a[n][n])
{
   float t[100];
   int k = 100;
   int i, j;
   for(i = 0; i <= 99; i += 1) {
      k = -i+99;
      t[-i+99] = a[i][i];
      for(j = 0; j <= 99; j += 1)
         a[i][j] /= t[-i+99];
   }
}
```

```
void compute(int p, int n, float a[n][n])
{
   float t[100];
   int k = 100;
   int i, j;
   //PIPS generated variable
   float __scalar__0;
#pragma omp parallel for private(k)
   for(i = 0; i <= 99; i += 1)
      k = -i+99;
#pragma omp parallel for private(j,__scalar__0)
   for(i = 0; i <= 99; i += 1) {
      __scalar__0 = a[i][i];
#pragma omp parallel for
      for(j = 0; j <= 99; j += 1)
         a[i][j] /= __scalar__0;
   }
}
```

- Let B and i be a loop body and index, and $W_{B,k}$ the k region function
- Let $R : Val \rightarrow \mathcal{P}(\Phi)\ s.t.\ R(v) = \{\phi | \exists \sigma\ \sigma(i) = v \wedge \phi \in W_{B,k}(\sigma)\}$
- If R is a mapping, k can be replaced by a scalar.

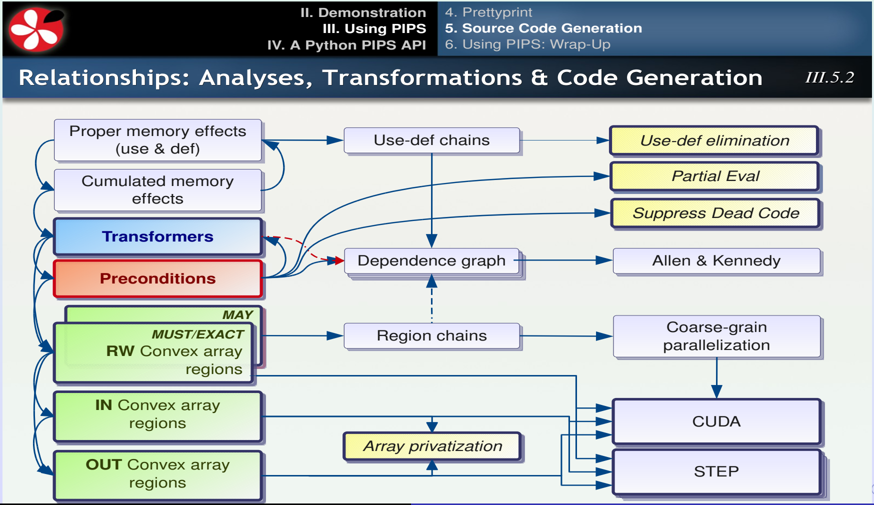
After dead code elimination

```
void compute(int p, int n, float a[n][n])
{
   float t[100];
   int k = 100;
   int i, j;
   //PIPS generated variable
   float __scalar__0;
#pragma omp parallel for private(k)
   for(i = 0; i <= 99; i += 1)
      k = -i+99;
#pragma omp parallel for private(j,__scalar__0)
   for(i = 0; i <= 99; i += 1) {
      __scalar__0 = a[i][i];
#pragma omp parallel for
      for(j = 0; j <= 99; j += 1)
         a[i][j] /= __scalar__0;
   }
}
```

```
void compute(int p, int n, float a[n][n])
{
   int i, j;
   //PIPS generated variable
   float __scalar__0;
#pragma omp parallel for private(j,__scalar__0)
   for(i = 0; i <= 99; i += 1) {
      __scalar__0 = a[i][i];
#pragma omp parallel for
      for(j = 0; j <= 99; j += 1)
         a[i][j] /= __scalar__0;
   }
}
```

A graph-based algorithm for a change!

The whole picture, almost...



II. Demonstration
III. Using PIPS
IV. A Python PIPS API

4. Prettyprint
5. Source Code Generation
6. Using PIPS: Wrap-Up

Relationships: Analyses, Transformations & Code Generation

III.5.2

Proper memory effects (use & def)
Cumulated memory effects
Transformers
Preconditions
MAY
MUST/EXACT
RW Convex array regions
IN Convex array regions
OUT Convex array regions
Use-def chains
Dependence graph
Region chains
Array privatization
Use-def elimination
Partial Eval
Suppress Dead Code
Allen & Kennedy
Coarse-grain parallelization
CUDA
STEP

Sample of passes applied

```
  SCALARIZATION                  updating   CODE(compute)
SCALARIZATION made for compute.
Request: build phase/rule PRIVATIZE_MODULE for module compute.
  PROPER_EFFECTS                 building   PROPER_EFFECTS(compute)
  CUMULATED_EFFECTS              building   CUMULATED_EFFECTS(compute)
  ATOMIC_CHAINS                  building   CHAINS(compute)
  PRIVATIZE_MODULE               updating   CODE(compute)
PRIVATIZE_MODULE made for compute.
Selecting rule: PRINT_PARALLELIZEDOMP_CODE
Module compute selected
Request: build resource PARALLELPRINTED_FILE for module compute.
  PROPER_EFFECTS                 building   PROPER_EFFECTS(compute)
  ATOMIC_CHAINS                  building   CHAINS(compute)
  CUMULATED_EFFECTS              building   CUMULATED_EFFECTS(compute)
  SUMMARY_EFFECTS                building   SUMMARY_EFFECTS(compute)
  INITIAL_PRECONDITION           building   INITIAL_PRECONDITION(compute)
  PROPER_EFFECTS                 building   PROPER_EFFECTS(main)
  CUMULATED_EFFECTS              building   CUMULATED_EFFECTS(main)
  SUMMARY_EFFECTS                building   SUMMARY_EFFECTS(main)
  INITIAL_PRECONDITION           building   INITIAL_PRECONDITION(main)
  PROGRAM_PRECONDITION           building   PROGRAM_PRECONDITION()
  TRANSFORMERS_INTER_FULL        building   TRANSFORMERS(compute)
  TRANSFORMERS_INTER_FULL        building   TRANSFORMERS(write_output)
  TRANSFORMERS_INTER_FULL        building   TRANSFORMERS(init_parameter)
  TRANSFORMERS_INTER_FULL        building   TRANSFORMERS(read_input)
  TRANSFORMERS_INTER_FULL        building   TRANSFORMERS(main)
  INTERPROCEDURAL_SUMMARY_PRECONDITION building   SUMMARY_PRECONDITION(main)
  PRECONDITIONS_INTER_FULL       building   PRECONDITIONS(main)
  INTERPROCEDURAL_SUMMARY_PRECONDITION building   SUMMARY_PRECONDITION(compute)
```

Approximate number of passes called

```
After database initialization: 0
After partial evaluation: 135
After parallelization: 158
After scalar privatization: 181
After array privatization: 255
After array scalarization: 328
After dead code elimination: 399
```

Conclusion: ACCA? YAKA? (sorry, French joke ☺)

- Difficulties hidden in a few analyses:
  T, P, W, R, IN, OUT
- Program transformations as simple[1] consequences of analyses?
  mapping, function, empty set,...
- Yes, sometimes
  Control simplification, contrant propagation, partial evaluation, induction variable substitution, privatization, scalarization, coarse grain loop parallelization,...
- But not always: graph algorithms are useful too
  Dead code elimination,...
- Transformations used to simplify analyses
  Compile to Analyze?

---

[1] Complexity?

Conclusion: genericity to simplify compilers?

- Analysis A depends on Analysis B, cycles are possible
- So abstract domain A depends on Abstract domain B
- A generic abstract domain interface such as APRON is not enough
- Simple domains are not useful with transformers
- Over-approximations are not sufficient: under-approximations or exact results are necessary for non-monotonous equations
- Do program transformations preserve abstract information? *Too much recomputing in PIPS* ☺

Questions?